**Activity 2 Worksheet (scaffolded): Minimising heat loss using differential calculus**

# Practice question 1

A small rectangular warehouse has a length equal to twice its width and is fully enclosed except for the floor. The roof loses twice as much heat per square metre as the walls. The warehouse has a volume of 2400 m3.

Find the dimensions of the warehouse that will minimise the total heat loss.

**Step 1: Define the variables.** (Remember the base of this house is a rectangle)

* Let be the width of the warehouse (in metres).
* The length is twice the width, so the length is (in metres).
* Let be the height of the warehouse (in metres).

The volume, , of the warehouse is 2400 m3 so:

Simplify:

Solve for :

**Step 2: Set up the heat loss equations.**

**Heat loss through the walls**

There are two walls of size , and two walls of size .

The total wall area, , in terms of is:

Substitute into the equation:

The heat loss through the walls is proportional to the wall area.

Use the heat loss equation for the wall:

**Heat loss through the roof**

The area of the roof (length × width) is

We are told that twice as much heat per square metre is lost through the roof as through the walls. So we know that

Therefore, the heat loss through the roof is:

**Step 3: Combine the heat loss equations to calculate total heat loss.**

Take out the common factor :

**Step 4: Use differentiation to find the point at which heat loss is a minimum.**

To minimise the heat loss, we need to differentiate the total heat loss function with respect to and set the derivative equal to 0.

We can ignore the constant since it does not affect the minimisation.   
Rewrite the function as

Then rewrite this with a negative power and differentiate:

**Step 5: Set the derivative equal to 0 to find the critical points, and solve the equation.**

Rearrange the equation to make the terms positive and solve:

**Step 6: Find the length and height.**

Remember, the length is twice the width.

Length = \_\_\_\_\_\_ m

Remember to use the formula for height you determined in step 1

Height = \_\_\_\_\_\_ m

**Step 7 Verify it’s a minimum.**

To verify that this is a minimum, check the second derivative of .

The second derivative of is:

Substitute in your value to the second derivative:

Since \_\_\_\_\_\_ > 0, the second derivative is \_\_\_\_\_\_, confirming that = \_\_\_\_\_\_ gives a minimum.

**Final answer**

The dimensions of the warehouse that minimise heat loss are approximately:

* Width = \_\_\_\_\_\_ m
* Length = \_\_\_\_\_\_ m
* Height = \_\_\_\_\_\_ m

# Practice question 2

A rectangular storage unit has a length three times its width and is fully enclosed except for the floor. The roof loses 50% more heat per square metre than the walls, but no heat is lost through the floor. The total volume of the storage unit is 3600 cubic metres.

Find the dimensions of the unit that will minimise the total heat loss, using the heat loss equation and differentiation.

**Step 1: Define the variables.**

Use these to determine the height of the house in terms of .

Write an equation for .

**Step 2: Set up the heat loss equations.**

**Heat loss through the walls**

There are two walls of size , and two walls of size .

So, the total wall area is:

**Heat loss through the roof**

The area of the roof is   
We know that 50% more heat per square metre is lost through the roof as through the walls.

Therefore, the heat loss through the roof is:

**Step 3: Combine the heat loss equations to calculate total heat loss.**

The total heat loss, , is the sum of the heat loss through the walls and the roof.

Take out the common factor :

**Step 4: Use differentiation to find the point at which heat loss is at a minimum**.

To minimise the heat loss, we need to differentiate the total heat loss function with respect to and set the derivative equal to 0.

We can ignore the constant since it does not affect the minimisation.

Rewrite the function:

Then rewrite this with a negative power and differentiate:

**Step 5: Set the derivative equal to zero to find the critical points, and solve the equation.**

**Step 6: Find the length and height.**

**Step 7: Verify it’s a minimum.**

Find the second derivative.

Substitute in the value of .

Explain how you know your answer is a minimum.

**Final answer**

The dimensions of the storage unit that minimise heat loss are approximately:

* Width = \_\_\_\_\_\_ m
* Length = \_\_\_\_\_\_ m
* Height = \_\_\_\_\_\_ m