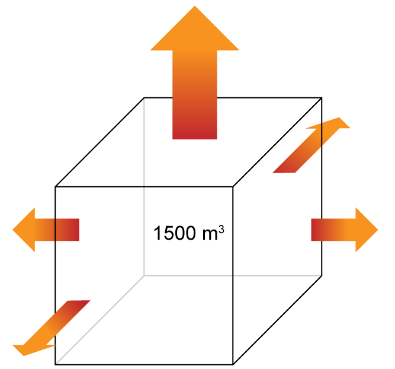
**Minimising heat loss using differential calculus**

# Worked example

A box-shaped house has a square floor. Three times as much heat per square metre is lost through the roof as through the walls, but no heat is lost through the floor.

If the house encloses 1500 m3, find the area of the floor that minimises heat loss and the height of the house.



**Remember:** The house is a cuboid. Its volume can be calculated by base × width × height.

**Step 1: Define the variables.**

* Let be the length of the base of the square floor in metres.
* Let be the height of the house in metres.

The volume of the house () is 1500 m3, so:

Solve for :

**Step 2: Set up the heat loss equations.**

The total heat loss comes from two components:

* heat lost through the **walls**;
* heat lost through the **roof**.

**Heat loss through the walls**

The house has four vertical walls. Each wall has an area of (base × height).   
Therefore, the total wall area () in terms of is:

We know that the heat loss through the walls is proportional to the wall area. We can use the heat loss equation for the wall:

**Heat loss through the roof**

The area of the roof is the same as the area of the floor.

The area of floor (base × base)

We are told that three times as much heat per square metre is lost through the roof as through the walls. This means we can find the U-value for the roof:

Therefore, the heat loss through the roof is:

**Step 3: Combine the heat loss equations to calculate total heat loss.**

The total heat loss, , is the sum of the heat loss through the walls and the roof:

Take out the common factors of *U*walls and from both terms:

**Step 4: Use differentiation to find the point at which heat loss is at a minimum.**

* To find the length of one side of the floor () which minimises heat loss, we need to find the critical points.
* We need to differentiate the total heat loss function, , with respect to and then set the derivative equal to zero.

We can now ignore the constant factor as this will not affect the minimisation.

Rewrite the function as:

First, rewrite the equation using a negative index.

Now differentiate with respect to .

**Remember:** When differentiating, we multiply the coefficient of by the index and then reduce the index by 1. For example: 4w3 becomes 12w2

**Step 5: Set the derivative equal to zero to find the critical points, and solve the equation.**

At the critical points , so:

Now, solve the equation. Start by rearranging to make any negative terms positive.

**Step 6: Find the height.**

Now that we know , we can find the height h using the volume equation:

**Step 7: Verify it’s a minimum.**

To verify that this is a minimum value, we can check the second derivative of .

**Remember:** If the second derivative is negative, the critical point will be a maximum.   
If the second derivative is positive, the critical point will be a minimum.

To find the second derivative, differentiate the first derivative.

**Remember:** When differentiating (which is equivalent to ), the index reduces to 0 to give . Anything to the power of 0 is equal to 1, so the derivative of is 1.

Substitute in the value of ,

Since 18 > 0, the second derivative is positive, confirming that gives a minimum.

**Final answer**

The area of the floor that minimises heat loss is and the height of the house is 15 metres.