Supporting Technical Education Teaching:

**Curriculum Resources**

Teaching Guide

Topic: Practical Construction Calculus

## Version information

|  |  |  |
| --- | --- | --- |
| **Version** | **Description of change** | **Date of Issue** |
| Version 1 | Original version | September 2025 |
|  |  |  |

|  |  |
| --- | --- |
| **Route** | Construction |
| **Qualification** | T Level Technical Qualification in Design, Surveying and Planning for Construction  <https://qualifications.pearson.com/en/qualifications/t-levels/design-surveying-and-planning-for-construction-2025.html> |
| **Topic** | Practical Construction Calculus |
| **Specification coverage** | **Content Area 7: Construction mathematical techniques**  7.2.1, 7.2.2, 7.2.3 |

This resource is part of a series of materials to support technical education teaching. The approach to developing the materials draws from research led by Professor Kevin Orr that sets out a model for understanding technical education pedagogy.

The curriculum development begins with the knowledge that students are working to learn and apply. Teachers draw from their subject and industry expertise, and their knowledge of their students, to make decisions about the core concepts the curriculum will focus on, how they will sequence these concepts and the activities that are selected to support students’ learning. The decisions behind the resources suggested in this topic are the result of choices made by the curriculum development team, which will be reviewed and improved by teachers’ decision-making and ongoing reflection in their own circumstances.

The materials also seek to support teachers in bringing classroom and industry closer together by providing assets that draw from authentic industry materials and using opportunities to capture workplace practice that can be shared with students.

HEALTH AND SAFETY

It is assumed that activities outlined in this Teaching Guide will be undertaken in suitable facilities or work areas and that good practices, appropriate use policies and procedures will be observed. Teachers should consult their employers’ risk assessments before use and consider whether any modification is necessary for the particular circumstances of their own class/institution.

HEALTH AND SAFETY

It is assumed that activities outlined in this Teaching Guide will be undertaken in suitable facilities or work areas and that good practices, appropriate use policies and procedures will be observed. Teachers should consult their employers’ risk assessments before use and consider whether any modification is necessary for the particular circumstances of their own class/institution.

HEALTH AND SAFETY

It is assumed that activities outlined in this Teaching Guide will be undertaken in suitable facilities or work areas and that good practices, appropriate use policies and procedures will be observed. Teachers should consult their employers’ risk assessments before use and consider whether any modification is necessary for the particular circumstances of their own class/institution.

HEALTH AND SAFETY

It is assumed that activities outlined in this Teaching Guide will be undertaken in suitable facilities or work areas and that good practices, appropriate use policies and procedures will be observed. Teachers should consult their employers’ risk assessments before use and consider whether any modification is necessary for the particular circumstances of their own class/institution.

Acknowledgements

We are grateful to the following individuals and organisations for their input: Dr Victoria Edmonson (author); Jo Locke (co-author); Taku Chirara (teacher advisor); Barry Lambert (curriculum advisor); Stephen Manley (industry advisor); Lee Hatwell (industry advisor); Balfour Beatty (industry advisors); MEI; Pearson.

Thank you to copyright holders for permission to use artwork, photographs and other copyrighted material: credits are included alongside their position in the materials. Every effort has been made to contact copyright holders of content reproduced in this topic.

T Level Technical Qualification is a qualification approved by IfATE.

Materials for other topics are available at: [www.technicaleducationnetworks.org.uk](http://www.technicaleducationnetworks.org.uk)

**Contents**

[**Introduction 5**](#_hcw3yx913w7j)

[Topic purpose 5](#_lqr9tfllkeiy)

[Industry importance 5](#_wu8qdiieyrqn)

[Industry links 6](#_68kwg1cgbbdq)

[Prior learning 7](#_p4g95ifqagpf)

[Accessibility 7](#_ei7b3hz8yxaj)

[Alternative scheme of work 8](#_ei7b3hz8yxaj)

[**Learning outcomes and specification coverage 10**](#_876lbvd6i7sg)

[**Lesson guidance 12**](#_tggfu4rk7tmn)

[Lesson 1: Using the mid-ordinate rule in construction 12](#_v4m7b395enob)

[*Preparation 12*](#_xo3v5jp9ycpg)

[*Activity guide 13*](#_pgdgu2cysq37)

[Lesson 2: Using the trapezoidal rule and Simpson’s rule in construction 18](#_esenlavx1v0f)

[*Preparation 18*](#_60vokwh0s7k5)

[*Activity guide 19*](#_7cvw9m802ewl)

[Lesson 3: Using differentiation in construction 24](#_dlhhj2becaxs)

[*Preparation 24*](#_lvo4dh2nf09e)

[*Activity guide 25*](#_cyvt5280z14l)

[Lesson 4: Using integration in construction 30](#_j8zcc5rbw5ca)

[*Preparation 30*](#_u8wloedqj5ts)

[*Activity guide 31*](#_9ywp9muxftnx)

[**Weblinks and resources 34**](#_gb3d9ehl3zht)

[**Terms of use and disclaimer of liability 36**](#_8ihh5f3hi4qh)

**Introduction**

This document for teachers outlines both the topic area covered and approaches to using the suite of resources and assets for each lesson. Unless otherwise stated, definitions of key terms have been developed by the authoring team and reviewed in the context of the activities. Teachers may choose to revise definitions as necessary and should review the content in advance of delivery to ensure it is appropriate for learners.

# Topic purpose

A strong understanding of a range of mathematical techniques is essential for students wishing to pursue a career in the construction sector. This topic is designed to enable students to develop an understanding of a range of mathematical techniques and their typical applications within construction scenarios. It focuses on the ability to select and apply basic differentiation and integration techniques correctly and understand how calculus is used to solve practical construction problems. As this topic has a mathematical focus, there is an expectation that all students have a sound mathematical understanding of GCSE maths, ideally at a higher level (see prior learning section for more detail).

This topic contains four lessons, each of which is assumed to be 1.5 hours. Each lesson is broken down to provide flexibility on the depth covered in the activities; lessons can also be split over multiple shorter lessons if required. Teachers may want to adapt the suggested sequencing of concepts and activities as appropriate for the students and circumstances.

Lessons 1 and 2 introduce the three rules (mid-ordinate, trapezoidal and Simpson’s) to estimate the area under the curve applying this to the real-life examples of building embankments and constructing new roads. Lessons 3 and 4 introduce students to the use of calculus and how it’s used in construction problems, including heat loss calculations.

Although the focus of this topic is on maths, there are also opportunities to build several essential skills that are developed during the course and general competencies for English and digital. These are listed in the section ‘Learning outcomes and specification coverage’.

# Industry importance

The mid-ordinate rule, trapezoidal rule, Simpson’s rule and integration are essential methods for estimating areas under curves, particularly when dealing with irregular shapes that are difficult to measure directly. In construction, these techniques are frequently used to calculate the cross-sectional areas of road embankments, trenches and channels, and for curved areas of buildings such as roofs. Accurate measurements from these calculations are used to determine the quantity of materials and then estimate costs. By using these mathematical methods, construction professionals can streamline their project planning and manage resources more effectively.

Differentiation is an important tool in calculating heat loss through building elements. Engineers can use rate of change principles from calculus to model how temperature gradients behave across different materials, helping to determine the efficiency of insulation and predict how heat will flow through walls, roofs and windows. This allows for more accurate design adjustments to minimise energy loss and improve overall building efficiency.

*“Maths is the backbone of construction, whether measuring distances or interpreting drawings. Maths skills are vital for project management, engineering and surveying.”*

***Nick Boyle, Technical Director at Balfour Beatty***

# Industry links

**General mathematic resources**

* BBC Bitesize (maths): clear explanations, revision notes and quizzes on various mathematical topics: [www.bbc.co.uk/bitesize/subjects/z38pycw](http://www.bbc.co.uk/bitesize/subjects/z38pycw)
* NRICH Maths: problem-solving challenges and activities suitable for learners aiming to stretch their mathematical thinking: [www.nrich.maths.org](http://www.nrich.maths.org)
* Desmos (graphing calculator): an interactive tool to create graphs and explore different functions; ideal for visual learners: [www.desmos.com](http://www.desmos.com)
* GeoGebra: dynamic mathematics software that brings together geometry, algebra and calculus, suitable for learners at various levels: [www.geogebra.org](http://www.geogebra.org)
* Oak National Academy: a source of material suitable for revising material from KS4: [www.thenational.academy/teachers/key-stages/ks4/subjects/maths/programmes](https://www.thenational.academy/teachers/key-stages/ks4/subjects/maths/programmes)
* LibreTexts: covers the mid-ordinate rule, trapezoidal rule and Simpson’s rule in a purely mathematical context: [math.libretexts.org/Courses/Mount\_Royal\_University/MATH\_2200%3A\_Calculus\_for\_Scientists\_II/2%3A\_Techniques\_of\_Integration/2.5%3A\_Numerical\_Integration\_-\_Midpoint%2C\_Trapezoid%2C\_Simpson%27s\_rule](https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2200%3A_Calculus_for_Scientists_II/2%3A_Techniques_of_Integration/2.5%3A_Numerical_Integration_-_Midpoint%2C_Trapezoid%2C_Simpson%27s_rule)

**Cut and fill calculation**

* SketchAndCalc: a useful guide on cut and fill calculation for construction, including tools to assist in volume estimation: [www.sketchandcalc.com](https://www.sketchandcalc.com/)
* Guide to cut and fill maps by Take-off Professionals: a detailed guide to calculating cut and fill volumes manually and using software; explanation on methods such as the cross-section and grid methods, along with more advance techniques – can be found online

**Heat loss and U-values**

* Designing Buildings: detailed insights into U-values, including how they are calculated and their significance in construction, energy-efficiency and building regulations: <https://www.designingbuildings.co.uk/U-values>
* Rockwool U-value Calculator: an interactive tool to calculate U-values for various construction materials and structure, particularly useful for insulation calculations; designed for construction stakeholders and helps assess thermal performance: [www.rockwool.com/uk/resources-and-tools/tools/u-value-calculator/](https://www.rockwool.com/uk/resources-and-tools/tools/u-value-calculator/)

# Prior learning

No prior learning of construction techniques is necessary to study the materials presented in this teaching guide. However, as this topic requires key mathematical skills used in the construction industry, students do need to be familiar with how to calculate the area, volume and perimeter of a range of 2D and 3D shapes. These are core concepts taught in GCSE maths, but some students may need to be reminded of how to perform these calculations. Students should also be confident with algebraic manipulation. All of these techniques are covered in 7.1.1–7.1.4 of the specification. It is assumed that these have been revisited, and students will have a clear understanding of these concepts before starting this topic.

Before teaching Lessons 3 and 4, it is essential that students are introduced to the concept of differentiation and integration. Most students will not have met this area of maths, as it is not covered in the GCSE Maths course. Students should be taught how to carry out one-step differentiation and integration, and the relevance of critical points to determine the minimum and maximum turning points on a curve. This should be taught initially as a purely mathematical skill. An alternative scheme of work that includes these skills is given below.

# Accessibility

The teaching materials have been designed to provide teachers with a flexible framework, including different approaches to activities, suggested consolidation activities to further embed knowledge and adaptable study questions to assess learning. As with all resources, teachers will wish to consider the specific needs of their students when using the materials, including Special Educational Needs and Disabilities (SEND). This will be particularly important for students who have dyscalculia. More information can be found on the Dyscalculia Association website. Although content has been reviewed, accessibility in externally linked resources cannot be guaranteed.

In some instances, two levels of worksheet are provided: ‘(scaffolded)’ files contain additional support, with a fully scaffolded approach to answering the first question and then decreasing levels of scaffolding for subsequent questions. Scaffolded worksheets should be used by students who require a greater degree of support with their mathematical understanding.

This series of lessons contains a lot of multi-step maths calculations. Many students may not be familiar with these calculations as some of the content is not covered at GCSE level. We have aimed to support students in mastering each of these concepts by:

1. including a detailed worked example;
2. including the same detailed worked example on a worksheet so students can follow through the steps and add any notes;
3. providing practice questions for students to complete using the same steps as the worked example;
4. providing detailed worked solutions to the practice questions. The fully worked solutions should enable students to work out where they may have gone wrong.

This is just one suggestion of the way the mathematical concepts could be presented. Teachers should choose how best to use these resources to meet the needs of their class. For example, the detailed worked solutions could also be used in 1-2-1 sessions or study periods.

Students’ base level of construction knowledge may affect how much explanation is needed to make appropriate industry links. Drawing on the students’ own experiences, including from their industry placements, will support this. Students’ understanding and experience of the world of work will be varied, which may mean they find applying the content to an industry environment challenging. Engaging in whole-group discussions where appropriate and sharing teachers’ and students’ own experiences will help expose students to different experiences. Visits from local people working in different areas of construction should be encouraged.

# Alternative scheme of work

The resources in this topic aim to contextualise specification objective 7.2 in the construction sector: they do not cover all of the underlying maths content. Therefore, teachers may prefer to teach the content in a longer block of six lessons, instead of four, to also introduce the maths content. A suggested route to do this, designed in consultation with maths specialists MEI, is given here. This builds on students' GCSE knowledge in its step-by-step approach. This builds on students’ GCSE knowledge in a step-by-step approach:

**Lesson 1: Calculate the area under a curve using the midpoint rule**

Use Gatsby TEN Construction: PCC Lesson 1. Full details of the lesson are provided on page 12 of this guide.

**Lesson 2: Calculate the area under a curve using the trapezoidal rule**

Use Gatsby TEN Construction: PCC Lesson 2. Full details of the lesson are provided on page 18 of this guide.

**Lesson 3: Differentiation of polynomials and use of differentiation for calculating**

This is an additional lesson to those provided in this series. In this lesson, the use of differentiation should be taught as a purely mathematical skill.

* Introduce the concept and rules for differentiation using software to demonstrate changing the gradient of a curve. (Note: there is no need for students to know how to derive results algebraically, but they do need to develop an awareness that the rules for differentiation make sense and are not arbitrary.)
* Introduce how to carry out one-step differentiation of polynomials.
* Explain the relevance of critical points to determine the minimum and maximum turning points on a curve.

These resources may prove useful when teaching this content:

* Calculus for Beginners, Chapters 0–7 by MIT Mathematics – can be found online
* HELM Workbooks 11–15: [nucinkis-lab.cc.ic.ac.uk/HELM/helm\_workbooks.html](https://nucinkis-lab.cc.ic.ac.uk/HELM/helm_workbooks.html)
* Gradient of a Curve video by Corbettmaths – can be found online
* Maths is Fun: This site could be used to introduce the basic concepts of derivatives and their role in calculus in a simple, visual way: [www.mathsisfun.com/calculus](https://www.mathsisfun.com/calculus/)
* Khan Academy (Differentiation Basics): This site offers comprehensive lessons on calculus, starting from the concept of a derivative to more advanced differentiation techniques. It includes video tutorials, practical exercises and interactive examples: [www.khanacademy.org/math/calculus-1](https://www.khanacademy.org/math/calculus-1)

(Please note: these online resources cover material beyond that which is required in the Construction T Level. You should refer to 7.2.1 and 7.2.2 in the specification to see which sections are needed.)

**Lesson 4: Use of differentiation in construction**

Use Gatsby TEN Construction: PCC Lesson 3. Full details of the lesson are provided on page 24 of this guide.

**Lesson 5: Integration of polynomials and use of integration to find area under a curve**

This is an additional lesson to those provided in this series. In this lesson, the use of integration should be taught as a purely mathematical skill.

* Introduce the concept of integration and that this technique is used to calculate the area under a curve on a graph.
* Introduce how to carry out one-step integration of polynomials.
* Explain the difference between definite and indefinite integration.
* Introduce how to use integration to calculate the area under a curve.

These resources may prove useful for teaching this content:

* Mathsisfun.com introduction to integration: [www.mathsisfun.com/calculus/integration-introduction.html](https://www.mathsisfun.com/calculus/integration-introduction.html)
* Khan Academy Integral Calculus: [www.khanacademy.org/math/integral-calculus/ic-integration](https://www.khanacademy.org/math/integral-calculus/ic-integration)
* Calculus for Beginners, Chapters 12–13 by MIT Mathematics – can be found online
* HELM Workbooks 11–15: [nucinkis-lab.cc.ic.ac.uk/HELM/helm\_workbooks.html](https://nucinkis-lab.cc.ic.ac.uk/HELM/helm_workbooks.html)

**Lesson 6: Use of integration in construction**

Use Gatsby TEN Construction: PCC Lesson 4. Full details of the lesson are provided on page 30 of this guide.

**Learning outcomes and specification coverage**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Lesson** | **Learning outcomes** | **Specification coverage** | **Skills and general competencies** | **Links to other specification content** |
| **1** | Students will be able to:   * explain the concept of cut and fill; * explain how an embankment is created and its purpose in construction; * estimate the area under a curve and apply this to a range of construction situations using the mid- ordinate rule. | **7.2.3** Numerical integration: (Simpson’s rule) mid-ordinate rule (trapezoidal rule) | General competencies  English:  **E2** Present information and ideas  **E5** Synthesise information  Maths:  **M2** Estimate, calculate and spot errors **M3** Work with proportion  **M4** Use rules and formulae  **M5** Process data  Digital:  **D3** Communicate and collaborate | **7.2.2** Integral calculus:   * Indefinite and definite integration techniques (one step) for polynomial and trigonometric functions * Constant of integration and initial conditions |
| **2** | Students will be able to:   * estimate the area under a curve and apply this to a range of construction situations using the trapezoidal rule; * estimate the area under a curve and apply this to a range of construction situations using Simpson’s rule; * compare the benefits and limitations of using the mid-ordinate, trapezoidal and Simpson’s rule in different situations. | **7.2.3** Numerical integration: (mid-ordinate rule), trapezoidal rule, Simpson’s rule | General competencies  English:  **E2** Present information and ideas  **E5** Synthesise information  Maths:  **M2** Estimate, calculate and spot errors  **M3** Work with proportion  **M4** Use rules and formulae  **M5** Process data  Digital:  **D3** Communicate and collaborate | **7.2.2** Integral calculus:   * Indefinite and definite integration techniques (one step) for polynomial and trigonometric functions * Constant of integration and initial conditions |
| **3** | Students will be able to:   * explain how heat is lost from a building and ways in which this can be minimised; * calculate U-values and heat loss through a structure; * use differential calculus to plan the construction of a house which minimises heat loss. | **7.2.1** Differential calculus: basic differentiation (one step) for polynomial and trigonometric functions | General competencies  English:  **E2** Present information and ideas  **E5** Synthesise information  Maths:  **M2** Estimate, calculate and spot errors  **M3** Work with proportion  **M4** Use rules and formulae  **M5** Process data  **M8** Communicate using maths  Digital:  **D3** Communicate and collaborate | **2.4.4** Causes of heat loss – through the fabric of the building, ventilation thermal bridging, air changes  **2.4.5** Factors affecting rates of heat loss – temperature differences, surface area, material heat transfer properties, air change rates  **2.4.6** Thermal conductivity and resistance – U-values, insulation materials  **2.4.7** Calculations involving thermal conductivity and resistance, heat loss. |
| **4** | Students will be able to:   * use integration to determine the area under a curve in various construction situations; * measure and analyse heat loss through a wall; * use integration to calculate the amount of heat lost through a wall in a specific time period. | **7.2.2** Integral calculus: indefinite and definite integration techniques (one step) for polynomial and trigonometric functions constant of integration and initial conditions | General competencies  English:  **E2** Present information and ideas  **E5** Synthesise information  Maths:  **M2** Estimate, calculate and spot errors  **M3** Work with proportion  **M4** Use rules and formulae  **M5** Process data  **M8** Communicate using maths  Digital:  **D3** Communicate and collaborate | **2.4.6** Thermal conductivity and resistance – U-values, insulation materials  **2.4.7** Calculations involving thermal conductivity and resistance, heat loss. |

**Lesson guidance**

# Lesson 1: Using the mid-ordinate rule in construction

This lesson introduces students to how earthworks are used in a road embankment and covers the concepts of cut and fill and the process of compaction. Students are shown how to use the mid-ordinate rule to calculate the quantity of earth materials needed in a specific situation. They will discuss the accuracy of using this technique.

The examples used in these lessons have been contextualised in the construction industry and chosen to provide appropriate opportunities to practice with the mathematical concepts involved.

You may wish to combine Lessons 1 and 2 and teach all three mathematical rules in one lesson.

## Preparation

|  |  |
| --- | --- |
| **Resources provided** | * L1 Slide deck * L1 Activity 2 Worked example * L1 Activity 2 Worksheet (scaffolded) * L1 Activity 2 Worksheet * L1 Activity 2 Worksheet answers * L1 Consolidation Worksheet |
| **Equipment needed** | Projector, whiteboard, worksheets, writing tools, graph paper, rulers, calculators, sample data set for mid-ordinate practice, internet access for research activity |
| **Prior learning** | * Students should be able to:   + calculate the area, volume and perimeter of a range of 2D and 3D shapes;   + plot and read co-ordinates on a graph. * Students should have covered 7.1.1–7.1.4 of the specification. |
| **Common misconceptions** | * The y-values at the end points (end co-ordinates) can always be used to calculate the mid y-values (mid-ordinates). In fact, the mid-ordinates are the y-coordinates of the point on the curve in the middle of an interval. * Unequal intervals can be used when dividing the curve into intervals. In fact, the intervals must always be of an equal width. Any interval can be used: smaller intervals increase accuracy but increase the amount of computation required. * The number of strips is always the same as the number of ordinates. This is the case for the mid-ordinate rule, but not for the trapezium or Simpson’s rule covered in Lesson 2. |
| **Accessibility** | * Consider using pair work to help students of different abilities to support one another. * Students’ prior maths knowledge may vary. Consider holding a drop-in session where students can have additional support in mastering essential maths skills. * Two levels of worksheet are provided with differing levels of support to perform calculations using the mid-ordinate rule. * Seek to ensure wide representation for any case studies used. * Basic animation is used in the slides in this lesson to improve cognitive load, stagger information or present instructions. Teachers may wish to remove this feature if it is unsuitable for students. |

## Activity guide

|  |  |
| --- | --- |
| **Introduction**  SUGGESTED TIME:  10 minutes  RESOURCES:   * L1 Slide deck – slides 2–4 | * Introduce the lesson objectives using slide 2. * Show students the image on slide 3 of workers excavating material at one part of the site and adding material to a different part of the site. * In small groups, ask students to discuss the answers to the three questions given on the slide. * Ask each group to feed back to the class, and share the definitions and descriptions on slide 4. Correct any misconceptions and fill any gaps in learning. |
| **Activity 1: Understanding embankments**  SUGGESTED TIME:  20 minutes  RESOURCES:   * L1 Slide deck – slides 5–10 | * This activity is designed to introduce students to the use of embankments and how they are built, and the use of the technique of ‘cut and fill’ in building a new road. * Show students the image of an embankment on slide 5. * In their same small groups from before, ask students to discuss the questions on the slide. * Show the video on embankments on slide 6 (<https://vimeo.com/1111914267/5caffc9b4b>), which explains what an embankment is and how one is constructed. As students watch the video, ask them to make brief notes to answer the questions shown on the slide. * Ask students to share their answers with the class. Ensure all students can explain the purpose and construction of an embankment. Example answers to the four questions are:  1. to support roads, railways or canals. They provide a stable foundation and allow the construction of these routeways over obstacles such as valleys, rivers and uneven terrain; 2. earth, gravels and clays arranged in layers; 3. to ensure that the embankment is built to the correct dimensions and shape. They are placed at intervals along the embankment and indicate the desired height and slope at each point along its length; 4. preparation, layering and compacting, using profile boards, final shaping of the embankment.  * A definition of an embankment and the main steps in its construction are summarised on slides 7 and 8. * Show students the image on slide 9 of a proposed longitudinal elevation for a new road over an area of terrain. Ask students to sketch the image on a whiteboard/piece of scrap paper and colour in the areas that they think should be cut and those that should be filled. Students can check their sketches with the diagram on slide 10. * Ask students to think of three reasons why it is important that the finished road is flat; for example, to provide suitable visibility for drivers, a smoother ride for users and to ensure proper drainage. |
| **Activity 2: Introduction to the mid-ordinate rule**  SUGGESTED TIME:  45 minutes  RESOURCES:   * L1 Slide deck – slides 11–27 * L1 Activity 2 Worksheet (scaffolded) * L1 Activity 2 Worksheet * L1 Activity 2 Worksheet answers * L1 Activity 2 Worked example * Calculator * Graph paper | * This activity will introduce students to the mid-ordinate rule. They will use this rule to estimate the area of a road embankment cross-section and determine how much material would need to be brought onto site. * Using slides 11–12, discuss with students the importance of correctly determining the amount of material which needs to be moved to create embankments, especially for large projects like HS2. More information about HS2 can be found at: [www.hs2.org.uk/building-hs2/cuttings-and-embankments/](https://www.hs2.org.uk/building-hs2/cuttings-and-embankments/) Teachers may want to include some information on this topic about HS2 from this link such as *The deepest cutting is the Lower Thorpe Cutting at 30m deep, and the longest runs from Barton to Mixbury at 4.1km (2.5 miles). Over 1.3 million cubic meters will be excavated here alone. That’s enough to fill the Royal Albert Hall over 10 times.* * Using slide 13, introduce the terms ordinate and mid-ordinate. * Using slide 14, introduce how the mid-ordinate rule is used to estimate the area under a curve.   + Explain that an ordinate is the y-coordinate of a point on a graph for a given x-coordinate. Here we are using ‘mid-ordinates’, which are the y-coordinates for the midpoints of the intervals.   + In this rule, the area of each interval is approximated by a rectangle with a height equal to the mid-ordinate.   + Explain that the midpoint is used as it balances out the part of the curve that is an underestimate and the part that is an overestimate. If an endpoint was used, the estimate would be less accurate. This addresses the first misconception that the endpoints can be used.   + Explain that the notation ‘yi’ is used to represent a general mid-ordinate. In specific examples, the ‘i’ is replaced with a number to represent each mid-ordinate in order. * Explain that in construction, the mid-ordinate rule is used to calculate cross-sectional areas of things like road embankments but also trenches and channels. Accurate measurements of these areas are important for planning how much material is needed and working out the associated costs. By using this rule, construction teams can plan their projects and manage materials more efficiently. * Explain that quantity surveyors use the mid-ordinate rule to estimate how much material is needed to be brought onto/shifted on site to create a suitable ground level on which a new road can be constructed. * Show slide 15, which describes how the mid-ordinate rule can be applied to cut and fill. Explain that “height above datum level” means the height above a fixed point that is used as datum level, such as a manhole cover. * Use the slides 16 to 24 and the Activity 2 Worked example document, which explains step by step how to complete this calculation. Students should follow through the worked example and raise any questions as the steps progress. * Use slide 25 to summarise the main steps from the worked example, which should be followed when using the mid-ordinate rule for a cut and fill problem. * Emphasise that the intervals should be equal and not too wide, as they need to account for changes in the shape of the curve. This addresses the second misconception that different interval widths can be used. * The more intervals used, the more accurate the estimate will be. * Use slide 26 to explain how these steps are summarised in the mathematical formula:   + Explain the notation that ‘delta x’ meaning ‘change in x’ is used to represent the interval width, and that is used to mean ‘approximately equal to’.   + The first version of the formula shows the sum of each rectangle area, and the second version simplifies this by taking out the common factor . * Display slide 27. Students practise using the mid-ordinate rule through completing the practice questions using Activity 2 Worksheet. There are two levels of the worksheet; one includes scaffolding which some students may find helpful, and one does not. * You may wish to point out that the cross-section of the embankment is across the width of the embankment (perpendicular to the direction of the road). A cross-section of a road would also be perpendicular to the direction of the road, but would show the road construction from formation level to finished surface and would be between the kerbs or edgings forming the limits of the road. The section along the length of the road would be known as a longitudinal section. * Draw students' attention to the fact that as embankments are an engineered structure, with sides that are sloped at a given angle and flat tops, when drawn in a cross-sectional diagram, their heights are joined with straight lines. Natural structures, such as a river channel, will have a more curved profile. * Share the fully worked solutions given on the Activity 2 Worksheet answer sheet so students can mark their own work. * Note: In the practice questions, the number of intervals is suggested, but if you provide additional questions for students (such as in the Consolidation activity), students may choose different interval widths, which may result in different estimates in the amount of material required. In these cases, ensure that the intervals are equal and that the scale chosen is appropriate. |
| **Plenary**  SUGGESTED TIME:  15 minutes  RESOURCES:   * L1 Slide deck – slides 28–29 | * Share with students the three reflection questions on slide 28. Students should discuss their answers in pairs before sharing with the class. * Possible discussion answers:  1. Dividing the base width into equal intervals ensures that each interval contributes proportionally to the overall area calculation. If the intervals were unequal, it would complicate the calculation process and likely result in inaccuracies because the contribution of each interval would vary, making it harder to ensure a balanced and correct estimate of the area. 2. The accuracy of the mid-ordinate rule is affected by the shape of the embankment. If the embankment has a relatively smooth and gentle curve, the mid-ordinate rule will provide a reasonably accurate estimate of the area. However, if the embankment has sharp changes in slope or irregular shape, the mid-ordinate rule may not capture these variations accurately, leading to a less accurate estimated area. The method assumes that each interval’s mid-ordinate represents the entire interval, which is less accurate for highly variable curves. 3. Examples are estimating the area of a road embankment or estimating the cross-sectional area of a river or channel.  * For question 3, a class list could be produced of possible situations where the mid-ordinate rule could be used, and this list could be used as a starting point for further research (see Consolidation task). * Review the responses provided by students and consolidate the learning from the lesson. * Finally, display the achieved learning outcomes on slide 29 to reinforce what has been covered in the lesson. |
| **Follow-up/ consolidation** (to be completed outside of lesson)  SUGGESTED TIME:  30 minutes  RESOURCES:   * L1 Slide deck – slide 30 * L1 Consolidation Worksheet * Calculator * Graph paper | * Ask students to complete one of two activities: the Consolidation Worksheet or a research task. * The Consolidation worksheet provides further practice calculating the area of material needed for other constructions, such as the volume of earth that needs to be dug out or filled in or designing reservoirs. The answers are:  1. 32.1 m2 2. 84.0 m2 3. 12.4 m2 4. 34.75 m2  * Note that answers may vary slightly due to the accuracy of reading the mid-ordinates from the graph. * Students may find it useful to use this resource to check their answers: [www.geogebra.org/m/WAKyrVkq](http://www.geogebra.org/m/WAKyrVkq) * For the research task, ask students to research:   + other situations where engineers would use the mid-ordinate rule and how this would be applied;   + other roles in the construction industry that may use the mid-ordinate rule. * Students could present their findings as a poster, including a worked example of the rule being used in practice. |

# Lesson 2: Using the trapezoidal rule and Simpson’s rule in construction

In this lesson, the trapezoidal rule and Simpson’s rule are introduced as further techniques to estimate the area under a curve. Both these rules require students to determine ordinates, building on the concept introduced by the mid-ordinate rule, and then input these into a mathematical formula.

Please note that throughout this and following lessons, the values read from graphs may not be exact and students’ readings may vary from those given in the lessons or answers.

Students will compare the mid-ordinate rule, which is covered in Lesson 1, and the two techniques from this lesson and their use in different construction situations.

If students are struggling with the new mathematical concepts, you may wish to split this lesson into two, focusing on one rule each lesson and allowing for additional time to practise each calculation.

## Preparation

|  |  |
| --- | --- |
| **Resources provided** | * L2 Slide deck * L2 Activity 1 Worked example * L2 Activity 1 Worksheet (scaffolded) * L2 Activity 1 Worksheet * L2 Activity 1 Worksheet answers * L2 Activity 2 Worked example * L2 Activity 2 Worksheet * L2 Activity 2 Worksheet answers * L2 Consolidation Worksheet * L2 Consolidation answers 1 * L2 Consolidation answers 2 |
| **Equipment needed** | Projector, whiteboard, worksheets, writing tools, graph paper, rulers, calculators |
| **Prior learning** | * How to use the mid-ordinate rule to calculate the area under a curve; this is covered in Lesson 1. * Students should be able to calculate the area of a trapezium. |
| **Common misconceptions** | * Unequal intervals can be used when dividing the curve into intervals. In fact, the intervals must always be of an equal width. Any interval can be used: smaller intervals increase accuracy but increase the amount of computation required. * The number of strips is always the same as the number of ordinates. In fact, when the ordinates used are the end-points of the intervals, there is always one more ordinate than the number of intervals. |
| **Accessibility** | * You may wish to use pair work to help students of different abilities to support one another. * Students’ prior maths knowledge may vary. Consider holding a drop-in session where students can have additional support in mastering essential maths skills such as calculating the area and volume of a range of 2D and 3D shapes (including trapeziums) and plotting co-ordinates. * Two levels of worksheet are provided with differing levels of support to perform calculations using the trapezoidal rule. * If you have less confident mathematicians, you may wish to split this lesson into two, focusing on one rule each lesson, allowing for additional time to practise each calculation. * Seek to ensure wide representation for any case studies used. * Basic animation is used in the slides in this lesson to improve cognitive load, stagger information or present instructions. Teachers may wish to remove this feature if it is unsuitable for students. |

## Activity guide

|  |  |
| --- | --- |
| **Introduction**  SUGGESTED TIME:  5 minutes  RESOURCES:   * L2 Slide deck – slides 2–6 | * Introduce the lesson by displaying the learning objectives on slide 2. * Show students slide 3, which lists the main steps of using the mid-ordinate rule, covered in Lesson 1, to calculate the area under a curve, but in the wrong order. Ask students to put the steps in the correct order. The answer is given on slide 4. * Recap the key areas where students could go wrong when using this rule. For example:   + not using equal intervals;   + not working out the ordinates of the midpoints;   + if calculating ‘cut’ and ‘fill’, not working out the height difference from the proposed new ground level. * Before moving on to Activity 1, you may wish to recap from Lesson 1. Ask students what they recall about the purpose of embankments and the concept of cut and fill. If necessary, show the summaries on slides 5 and 6. |
| **Activity 1: Introduction to the trapezoidal rule**  SUGGESTED TIME:  40 minutes  RESOURCES:   * L2 Slide deck – slides 7–20 * L2 Activity 1 Worked example * L2 Activity 1 Worksheet (scaffolded) * L2 Activity 1 Worksheet * L2 Activity 1 Worksheet answers * Calculator * Graph paper | * This activity will introduce students to the trapezoidal rule. They will then use this to estimate the area of a road embankment cross-section and determine how much material would need to be brought onto site. * Explain that there are two more rules that can be used to estimate the area under a curve: the trapezoidal rule and Simpson’s rule. Similar to the mid-ordinate rule, they can be used to determine the amount of material needed to be moved or excavated when constructing embankments, roads or tunnels as well as calculating the area beneath a curved roof or window. * Use slide 7 to introduce the trapezoidal rule. It may be useful to mention that this is sometimes referred to as the trapezium rule. * Remind students how the area of a trapezium is calculated using slide 8 before introducing them to the trapezoidal rule formula on slide 9. * Describe how the formula is derived:   + If the width of the curve is split into intervals, then there will be ordinates: to . Highlight that the number of ordinates will always be one more than the number of intervals. This addresses the misconception that the number of intervals and ordinates are the same.   + Remind students that the ordinate refers to the y-coordinate of a point on the graph. Highlight that the x-coordinates used to determine the ordinates in this rule are at the edges of the intervals, not the midpoints.   + Each ordinate is the height of the parallel side of a trapezium.   + For the trapezoidal rule, we simply sum the areas of the trapezium.   + The formula can be simplified by first taking out the common factor (half the width of the trapeziums), and then by collecting the like terms. There are two copies of each ordinate, except for the first and last ( and ).   + Explain that the sign means ‘sum’ and this portion of the formula means ‘the sum of the middle ordinates ( to ) all multiplied by 2’. * Students are more likely to remember this formula in simpler terms: * Explain that, at first, a formula like this can appear difficult to understand, but we can break it down into a series of steps. * Show slide 10 and ask students to work out the ordinate of y3 (2 m) and y4 (1.5 m). * Explain the worked example using slides 11–17. Students should follow along using the Activity 1 Worked example document, which explains step by step how to complete this calculation; students should raise any questions as the steps progress. * Use slide 18 to introduce the steps which should be followed when using the trapezoidal rule to calculate the area under a curve. * Emphasise that when using the trapezoidal rule, while there can be any number of intervals, they must be an equal width. Mention that the sections should not be too large, as they need to account for changes in the shape of the curve. * Explain that one main difference when using this rule compared to the mid-ordinate rule is that the interval endpoints are used to determine the ordinates rather than the midpoints. * Display slide 19. Students practise using the trapezoidal rule through completing the practice questions on Activity 1 Worksheet. There are two levels of the worksheet; one includes scaffolding which some students may find helpful, and one does not. * Hand out a copy of the Activity 1 Worksheet answers so students can mark their own work. * Remind students that the trapezoidal rule approximates the area by dividing the area under an embankment or road into trapezoids, using straight lines to connect the points along the curve. This method is simple but tends to be less accurate than other methods, especially for curves with significant curvature, because straight lines do not capture a curve's shape well. * Share with students the three reflection questions on slide 20. In pairs, students should discuss their thoughts before sharing with the class. * Possible discussion answers:  1. As with the mid-ordinate rule, dividing the base width into equal intervals ensures that each interval contributes proportionally to the overall area calculated. The same reasoning presented in Lesson 1 for reflection question 1 can be applied here. 2. The accuracy of the trapezoidal rule depends on how straight or curved the embankment’s shape is. If the embankment has a smooth, straight slope, the trapezia will approximate the area well. However, if the embankment has sharp curves or irregular shapes, the straight-line approximation of the trapezia may lead to overestimating or underestimating the actual area. 3. The trapezoidal rule can be useful in various real-world applications, such as estimating the area of irregularly shaped fields, calculating the volume of liquid in tanks with curved walls. |
| **Activity 2: Introduction to Simpson’s rule**  SUGGESTED TIME:  35 minutes  RESOURCES:   * L2 Slide deck – slides 21–34 * L2 Activity 2 Worked example * L2 Activity 2 Worksheet * L2 Activity 2 Worksheet answers * Calculator * Graph paper | * This activity will introduce students to Simpson’s rule. * Explain that in contrast to the trapezoidal rule, Simpson's rule approximates the area under a curve by fitting parabolic arcs through the intervals. * Use slide 21 to remind students that a parabolic arc is the shape of a quadratic curve. Using parabolic arcs provides a more accurate approximation for most curves, as parabolas better match the shape of smooth, curved lines. * Slide 22 gives examples of places where parabolic curves are seen in real life, such as the path of water from a fountain, the shape of certain bridges or even a roof. As a class, discuss why they are commonly chosen in architecture, both in terms of aesthetics and strength. * Using slide 23, introduce Simpson’s rule and how it is used to estimate the area under a curve. * A key difference between Simpson’s rule and the trapezoidal rule is that Simpson’s rule requires an even number of intervals (and therefore an odd number of ordinates), whereas the trapezoidal rule can work with any number of intervals (odd or even) as long as they are of equal width. The trapezoidal rule is easier to apply, but Simpson's rule is typically more accurate for curved shapes. * Show the mathematical formula and the simpler word version on slide 24. Explain that while it can look complicated, it can be broken down into a number of steps. * Explain the worked example given on slides 25–31. Students should use the Activity 2 Worked example document to follow along and raise any questions as the steps progress. * Use slide 32 to introduce the steps which should be followed when using Simpson's rule to calculate the area under a curve. * You may wish to point out that in the ‘real world’, the buyer or quantity surveyor would normally work from the elevation drawings produced by the architect and the dimensions would be taken/scaled from these. * Like the other techniques, Simpson’s rule could be used to calculate the area of an embankment, for example, along a riverbank, but in construction, these structures are normally linear rather than curved. * Show students the discussion questions on slide 33. Give them a couple of minutes to discuss their answers in pairs before sharing their thoughts with the class. * Possible discussion points:   + Where shapes have a straight or a linear profile with very little curvature (such as in this road embankment), the straight line approximation of the trapezoidal rule is more accurate.   + Simpson's rule is more accurate for curved shapes, such as a curved roof line. When deciding which rule to use, you should think about the shapes produced when the cross-section is divided into intervals as well as the overall shape. * Display slide 34. Students practise using Simpson’s rule through completing the practice questions on the Activity 2 Worksheet in two different construction settings. * Hand out the Activity 2 Worksheet answers so students can mark their own work. You may wish to discuss how much material a company is likely to order based on the approximation of an area calculated. For example, in question 2, you will not be able to purchase exactly 251.92 m2 of cladding. * This may be an opportunity to link to the calculation of all-in rates, where any material costs factor in a wastage percentage. Explain that more cladding would be ordered than the exact amount calculated to allow for breakage or other issues when fitting. |
| **Plenary**  SUGGESTED TIME:  10 minutes  RESOURCES:   * L2 Slide deck – slides 35–37 | * This activity is designed to compare each of the three rules for area approximations: the mid-ordinate rule, the trapezoidal rule and Simpson’s rule. * Show the table on slide 35. For each statement, ask students to determine which of the rules it applies to. Some statements apply to more than one rule. The answers are provided on slide 36. Correct any misconceptions that may have come up in the discussion. * Use slide 37 to consolidate the learning from the lesson. Ensure students are clear about the link between the mathematical rules and their use in the construction industry. |
| **Follow-up/ consolidation** (to be completed outside of lesson)  SUGGESTED TIME:  40 minutes  RESOURCES:   * L2 Slide deck – slide 38 * L2 Consolidation Worksheet * L2 Consolidation answers 1 * L2 Consolidation answers 2 * Calculator * Graph paper | * Students should read the case study described on the Consolidation Worksheet and on slide 38. They will use the mid-ordinate rule, trapezoidal rule and Simpson’s rule to calculate the area of cut and fill for a road and compare the results. They will explain any differences in the calculated areas and why those differences might have occurred. * When introducing the consolidation task, you may wish to remind students what the formation level of a road is: the bottom layer of the road structure, prepared and levelled to support the overlying layers, which can be made of bituminous materials or concrete. * Students may wish to look back at the worked examples they have been provided with for each of the three rules (Worksheets: L1 Activity 2, L2 Activity 1 and L2 Activity 2) to help them. They may also find this resource helpful to check their answers and compare the rules: [www.geogebra.org/m/WAKyrVkq](https://www.geogebra.org/m/WAKyrVkq) * It is intended that the activity will be self-marked by students; however, depending on the lesson time available, this could be used as a teacher-led task. * Detailed solutions are provided on the answer sheets. Students should use Answer sheet 1 to check their answers to exercise 1 before moving on to exercise 2. * In exercise 2, they should use the values provided on Consolidation answers 1 where the contour lines have been rounded to multiples of 2.5 to make calculations easier. |

## 

# Lesson 3: Using differentiation in construction

This lesson covers identifying areas of heat loss from a house using thermal camera images and how this can be prevented through different types of insulation. Students will learn about the factors that affect heat loss through a wall and derive the formula for calculating heat loss. Students will practise calculating the U-value of different building components to analyse the heat loss through them.

This lesson also introduces how differentiation calculations can be used to plan the construction of a house that minimises heat loss. If following the alternative scheme of work, this lesson should follow after the lesson covering differentiation of polynomials, as purely a mathematical skill.

## Preparation

|  |  |
| --- | --- |
| **Resources** **provided** | * L3 Slide deck * L3 Activity 1 Worked example * L3 Activity 1 Worksheet * L3 Activity 2 Worked example * L3 Activity 2 Worksheet (scaffolded) * L3 Activity 2 Worksheet * L3 Activity 2 Worksheet answers * L3 Consolidation Worksheet 1 * L3 Consolidation Worksheet 2 * L3 Consolidation 2 answers |
| **Equipment** **needed** | Projector, whiteboard, worksheets, writing tools, calculators |
| **Prior** **learning** | * From GCSE, students should be familiar with:   + calculating the area, volume and perimeter of 2D and 3D shapes;   + algebraic manipulation. * Students should have covered 7.1.1–7.1.4 of the specification. * It is essential that students have been introduced to the concept of differentiation. Students should be able to carry out one-step differentiation and understand the relevance of critical points to determine the minimum and maximum turning points on a curve. |
| **Common misconceptions** | * The higher the U-value, the better a product is at stopping heat escaping. In fact, a lower U-value indicates that the product is better at preventing heat moving through it. * Belief that the °C and K units are not the same size, i.e. assuming a temperature difference of 10°C is not the same as a temperature difference of 10 K. In fact, a change of 1°C is equivalent to a change of 1 K. * Differentiation is simply dividing terms rather than finding the rate of change of a function. * Differentiation is purely a mathematical operation. Not understanding that the derivative represents the gradient of the tangent line at a point on a curve. * Incorrectly applying the power rule when differentiating; most commonly, failing to subtract 1 from the exponent after multiplying, for instance, differentiating x2 as 2x2 instead of 2x. |
| **Accessibility** | * Consider using pair work to help students of different abilities to support one another. * Students’ prior maths knowledge may vary. Consider holding a drop-in session where students can have additional support in mastering essential maths skills. * When finding minimum values for functions, some students may find it hard to visualise what is going on. Using graph drawing software such as [www.desmos.com/calculator](https://www.desmos.com/calculator) or [www.geogebra.org/calculator](https://www.geogebra.org/calculator) can be helpful. * Two levels of the Activity 2 Worksheet are provided with differing levels of support to perform the heat loss calculation using differentiation. Worksheet 1 is fully scaffolded. * An alternative consolidation activity (Consolidation Worksheet 2) is provided, which offers students – struggling with differentiation – an opportunity to practise more differentiation calculations. * Fully worked solutions are provided so students can mark their own work. * Seek to ensure wide representation for any case studies used. |

## Activity guide

|  |  |
| --- | --- |
| **Introduction**  SUGGESTED TIME:  15 minutes  RESOURCES:   * L3 Slide deck – slides 2–5 | * Introduce the lesson using the learning objectives on slide 2. * Show the image of a thermal imaging camera capturing heat loss from a building on slide 3. * As a class, discuss the main areas where heat is lost from a house and how the heat loss could be reduced, and create a class list. Example answers include:   + draughts around windows or doors: use draft excluders;   + through the roof: add fibre glass insulation;   + through the windows: fit double-glazed windows. * Discuss why preventing heat loss is important. For example:   + to prevent energy being wasted;   + to reduce heating costs;   + to protect the environment: by reducing the volume of greenhouse gases released when fuels are burnt and the cost to the environment of extracting coal;   + to improve sustainability. * Explain that heat loss through walls is one of the largest contributors to overall heat loss. Walls act as barriers to keep the warmth inside, but heat can still pass through them because heat always tries to move from warmer places to colder places. Whenever the air inside a building is warmer than outside, the heat will try to escape through the walls. * Use slide 4 to explain how heat can be lost through conduction, and radiation. Highlight that good insulation is key to reducing all these types of heat loss through walls. * Show slide 5. Explain that a thermal imaging camera is a special camera that shows temperature using colours. They are used to identify if an area is hot or cold, even if you can’t feel it. These cameras can be used to check if heat is escaping from a building. The colours represent different temperatures, making it easy to see where heat is being lost or where something is warm. * Ask students to look at the image and work out where most heat is escaping from.   + They should notice that most heat appears to be escaping through the walls.   + The windows are coloured blue, which suggests that they may be double- or triple-glazed as very little heat is being lost through them. * Explain that cameras like this can be used to check that a material is still working efficiently. For example, over time, the seal on double-glazing can fail and needs to be replaced. * If time allows, and you have the equipment available, students could use a digital thermal imaging camera to look for the areas of greatest heat loss in a building that they have access to, and then consider why heat is lost from those areas. |
| **Activity 1: Calculating U-values and heat loss through a wall**  SUGGESTED TIME:  30 minutes  RESOURCES:   * L3 Slide deck – slides 6–21 * L3 Activity 1 Worked example * L3 Activity 1 Worksheet | * This activity will introduce students to how to calculate U-values and heat loss through a wall. Using the prompts on slide 6, ask students to make a list of the factors that increase and decrease the rate of heat loss.   + Increase rate of heat transfer by increasing the area of contact, decreasing the thickness of the wall, increasing the temperature differential, using a material with poorer insulating properties.   + Decrease the rate of heat transfer by decreasing the area of contact, increasing the thickness of the wall, decreasing the temperature differential, using a material with better insulating properties. * Introduce the formula using slide 7. Make sure the students understand what each variable represents. * Explain thermal resistance and the factors that it depends on using slide 8, and then use slide 9 to explain how it relates to the heat loss formula. * Before introducing students to U-values, use slide 10 to remind them about the temperature scales and how 1°C temperature change is equal to a 1 K temperature change. It is a common misconception that these temperature changes aren’t equivalent, so ensure that students have understood this. * Use slides 11–13 to explain U-values. Describe how a U-value is related to the thermal resistance. If necessary, remind students that the reciprocal of a value is equal to 1 divided by that value. Example U-values are shown on slide 13. * Remind students that a low U-value will keep more heat inside a building, helping to save energy and keep the space warm. This helps tackle the misconception that a higher U-value is ‘better’ than a lower U-value. * Use slides 14–15 to describe how a U-value is calculated. Highlight that the layers for a building element can also include fixings and air gaps. * Explain to students that the overall U-value of a building (calculated by combining the U-value of its different components) is extremely important. There are certain standards that must be reached according to Building Regulations. One reason for this is reducing the environmental impact a building has as energy is saved through unwanted heat loss. * Use slide 16 to explain that when planning the construction of a new building, discussions must take place between designers, clients and architects to ensure it is practical, achievable and meets building requirements, such as fire protection and durability. Use the example of RAAC (reinforced autoclaved aerated concrete) given on the slide. * Show slide 17 and demonstrate how the heat loss formula can be transformed using U to give the common formula for heat loss used by the construction industry. * Use slides 18–19 to work through a step-by-step example calculating a U-value and the resulting heat loss. Students can follow the example on the Activity 1 Worked example handout. * Explain that a typical cavity wall consists of two layers (or leaves) of masonry with a gap (cavity) between them. The materials used in each layer and the cavity itself have different thermal resistances. * Standard thermal resistance values are listed in reference tables (the values students need are provided on the worksheet). * Display slide 20. Students should use the Activity 1 Worksheet to calculate U-values and heat loss. * Share the answers on slide 21 so students can mark their own work. * Note that correct answers might vary depending on whether students round during the calculation or keep the U figure in their calculator. The answers on the slide represent answers when rounded during the calculation. If students keep the figure in their calculators at each stage, they will get 83.6 (rather than 83.5) in exercise 1 and 41.39 (rather than 41.375) in exercise 2. * Explain to students that to enable the government to achieve its legal target for net zero greenhouse gas emissions by 2050, minimum U-value requirements for buildings have been increased for both new building and improvements to existing buildings: [www.bwf.org.uk/wp-content/uploads/Future-Home-Standards-U-Value-guidance-document-March-21.pdf](https://www.bwf.org.uk/wp-content/uploads/Future-Home-Standards-U-Value-guidance-document-March-21.pdf) |
| **Activity 2: Minimising heat loss using differential calculus**  SUGGESTED TIME:  35 minutes  RESOURCES:   * L3 Slide deck – slide 22 * L3 Activity 2 Worked example * L3 Activity 2 Worksheet (scaffolded) * L3 Activity 2 Worksheet * L3 Activity 2 Worksheet answers | * This activity will introduce students to how to calculate heat loss using differential calculus. * Explain that we can use heat loss calculations to help us optimise the design of buildings. * Display slide 22. Using the Activity 2 Worked example document, run through the steps on how to complete a minimising heat loss calculation. Ensure that students follow along with the calculations and tackle any misconceptions with the process and purpose of differentiation that arise. * Discuss that, in the example, the house is a cuboid. As the roof and floor size are the same, the smaller the floor (and therefore, the roof), the less heat is lost from the building since most heat is lost through the roof. * Give students Activity 2 Worksheet that accompanies this activity. There are two levels of the worksheet; one includes scaffolding which some students may find helpful, and one does not. * Students practise using differential calculus to minimise heat loss through completing the practice questions on the worksheet. * Hand out Activity 2 Worksheet answers to students so they can mark their own work. Alternatively, you may wish to stop students regularly (e.g. after each step) so you can identify any areas where students are struggling and provide further guidance. |
| **Plenary**  SUGGESTED TIME:  10 minutes  RESOURCES:   * L3 Slide deck – slides 23–29 | * Display slide 23. Discuss with students how U-value and heat loss calculations directly affect decision-making in construction projects (e.g. cost savings, sustainability). Possible points for discussion:   + Information about U-values and heat loss is used to choose materials that keep buildings warmer in winter and cooler in summer.   + Good insulation and energy-efficient designs mean a building uses less energy for heating and cooling. This can reduce energy bills and ensures the building is cheaper to run over its lifetime.   + Good insulation and energy-efficient designs help the environment by lowering energy use; this increases sustainability.   + By doing these calculations, builders can make better choices to create buildings that are comfortable, cost-effective and good for the planet. * Show students the multiple-choice questions on slides 24–27 and give them one minute to answer each question. Review the responses given and consolidate the learning from the lesson using slide 28. * Lesson 4 contains an activity on heat loss which requires measurements to be taken prior to the lesson. If you wish students to complete this before the next lesson, display slide 29, otherwise this can be skipped. Explain to students that they will need to take temperature measurements at 20-minute intervals inside and outside a wall over a period of 1–2 hours. They will use the data in an integration activity to calculate the total heat loss. |
| **Follow-up/ consolidation** (to be completed outside of lesson)  SUGGESTED TIME:  30 minutes  RESOURCES:   * L3 Slide deck – slide 30 * L3 Consolidation Worksheet 1 * L3 Consolidation Worksheet 2 * L3 Consolidation Worksheet 2 answers | * Display slide 30. Give each student a copy of Consolidation  Worksheet 1. They should research three different insulation materials and compare the materials based on their U-value, cost and environmental impact, summarising their findings in a table. * Alternatively, Consolidation Worksheet 2 can be used to give students an opportunity to practise more calculations involving differentiation. There is a worked example of how to calculate the minimum number of fencing panels required to enclose a construction site, and two practice questions. Fully worked solutions are available on the Consolidation Worksheet 2 answers handout so students can mark their own work. |

# Lesson 4: Using integration in construction

This lesson provides an introduction to how integral calculus is used in a range of different construction settings. Students will use integration to find the area under a graph to determine the amount of heat that is lost in a specific time period through a wall in terms of watts per hour.

If following the alternative scheme of work, this lesson should follow after Lesson 5, which covers integration of polynomials as purely a mathematical skill.

Note that Activity 2 in this lesson requires temperature measurements to be taken every 20 minutes over the course of 1.5–2 hours. This could be done before the lesson.

## Preparation

|  |  |
| --- | --- |
| **Resources provided** | * L4 Slide deck * L4 Activity 1 Worked example * L4 Activity 1 Worksheet (scaffolded) * L4 Activity 1 Worksheet * L4 Activity 1 Worksheet answers * L4 Activity 2 Worksheet * L4 Consolidation Worksheet * L4 Consolidation Worksheet answers |
| **Equipment needed** | Projector, whiteboard, worksheets, writing tools, calculators, thermometer, computers/tablets with access to Excel |
| **Prior learning** | * Students should be familiar with:   + calculating the area, volume and perimeter of a range of 2D and 3D shapes;   + algebraic manipulation;   + finding the equation of a line (*y* = *mx* + *c*). * Students should have covered 7.1.1–7.1.4 of the specification. * Students should be familiar with the heat loss equation (introduced in Lesson 3). * It is essential that students have been introduced to the concept of integration and how to carry out one-step integration. |
| **Common misconceptions** | * Mixing up integration and differentiation rules; this is particularly common with polynomial functions (functions involving powers). |
| **Accessibility** | * When completing the group task, consider grouping students of different abilities to support one another. * Students’ prior maths knowledge may vary. Consider holding a drop-in session where students can have additional support in mastering essential maths skills. * For Activity 1, two levels of worksheet for are provided with Worksheet 1 providing additional support with the calculations. * An alternative consolidation activity is provided which offers an opportunity for students who find the concept of integration difficult to practise more integration calculations. Answers are provided so students can mark their own work. * Seek to ensure wide representation for any case studies used. * Basic animation is used in the slides in this lesson to improve cognitive load, stagger information or present instructions. Teachers may wish to remove this feature if it is unsuitable for students. |

## Activity guide

|  |  |
| --- | --- |
| **Introduction**  SUGGESTED TIME:  10 minutes  RESOURCES:   * L4 Slide deck – slides 2–3 | * Introduce the lesson using the learning objectives on slide 2. * Show the video on slide 3 (<https://vimeo.com/1111914156/70b6d7679b>) which explains the different ways calculus can be used. Explain that this lesson will focus on two different uses of integration in construction: calculating the area under a curve and calculating the total amount of heat loss through a wall over a period of time. * Once students have watched the video, assess their understanding by asking them some questions and allow students to ask any questions that they may have. |
| **Activity 1: Using integration to calculate the area under a curve**  SUGGESTED TIME:  35 minutes  RESOURCES:   * L4 Slide deck – slides 4–9 * L4 Activity 1 Worked example * L4 Activity 1 Worksheet (scaffolded) * L4 Activity 1 Worksheet * L4 Activity 1 Worksheet answers * Calculator | * This activity introduces how to use integration to calculate the area under a curve. * If Lesson 2 of this sequence has been taught, remind students that they used Simpson’s rule to estimate the area under a curve to solve construction problems. In this activity, they will learn how integration can be used to solve similar problems. * Some students may struggle to connect integration with finding an area under a curve. Explain that integration is equivalent to finding the sum of an infinite number of slices of the curve to find the total area. * Using slide 4, introduce how integration is used to calculate the area under a curved roof. Students should follow along with the Activity 1 Worked example worksheet, which explains step by step how to complete this calculation. * Use slides 5–8 to work through the step-by-step Integration example. * Students practise calculating the area under a curve in different construction situations through completing the practice questions on Activity 1 Worksheet. There are two levels of the worksheet; one includes scaffolding which some students may find helpful, and one does not. * Display slide 9. Hand out copies of the Activity 1 Worksheet answers so students can mark their own work. Alternatively, you may wish to stop students regularly (for example, after each step) so you can identify any areas where students are struggling and provide further guidance. This would be a good opportunity to check for any misconceptions, such as confusing the processes of differentiation and integration. |
| **Activity 2: Measuring and analysing heat loss through a wall**  SUGGESTED TIME:  40 minutes  RESOURCES:   * L4 Slide deck – slides 10–22 * L4 Activity 2 Worksheet * Thermometers * Excel spreadsheet | * Explain to students that now they can apply the principles of integration to another construction situation: determining the total amount of heat loss through a wall. * This activity is best done on a day when there is a clear temperature difference between the inside and outside. For this activity, students need to measure the temperature every 20 minutes for 1.5–2 hours, so this needs to occur throughout the lesson or have been completed prior to the session. The suggested activity time is for the task introduction and for students to complete the practical write-up. * Give students the Activity 2 Worksheet. In this practical activity (show slide 10), they will:   + assess the form and date of construction of a wall;   + work in pairs to measure and record the temperature using thermometers on both the inside and outside of their chosen wall at different time intervals throughout the day. They should use the table on the worksheet to record their results;   + analyse the data using an Excel spreadsheet. * The goal is to plot the temperature differences graphically, use Excel to generate a formula for the best fit line and then integrate under the curve to determine the total heat loss over time.   Slides 11–22:  Before the students carry out/write up their practical, use these slides to talk through the worked example (slide 11) of how to perform the heat loss calculation. Correct answers can vary slightly depending on whether students round during their calculation or keep figures in a calculator for the whole calculation.   * Show the sample data on slide 12. Please note this data produces a straight line graph and represents a more sudden increase in the outside temperature than students are likely to find in the UK. In reality, heat loss would normally produce an exponential curve, as heat loss is high at large temperature differences and low at small temperature differences. * It’s possible that over a couple of hours, the temperature difference may be fairly constant. If this is the case, focus on using the sample data and make sure students understand this worked example. * Show the diagram on slide 13 to illustrate how the temperature changes through a boundary, such as a wall. * Show the graph on slide 14 which graphs the temperature data given in the example. Mention that in this particular example, the points roughly form a line. * To illustrate how the equation of the line can be calculated manually, show the worked example on slides 15–18; however, this step can be skipped as software such as Excel can be used to generate the equation of the line. * Explain that the equation of the line can be calculated by entering the data into a spreadsheet and asking the software to draw a graph and determine the line of best fit. In Excel, this is done by creating a scatter graph, selecting a linear trendline and choosing the option to show the equation. * Use slides 19–20 to explain how to set up the integral for heat loss and perform the integration. Explain that to solve the problem using normal integration, we need a continuous function for the temperature difference (Δ*T*). In this case, we are given temperature data at discrete intervals, so to apply integration, we can approximate the change in temperature difference over time as a linear function. To do this, we:   + draw a line that best approximates the data (line of best fit);   + find the equation of this line in the form *y* = *mx* + *c*;   + integrate the linear function;   + calculate total heat loss. * The final step on slides 21–22 is to calculate the total heat loss using the heat loss equation. Explain to students that heat loss is normally expressed in terms of watts per hour, so they need to convert their final answer from watts per minute to watts per hour.   Remind students that the amount of heat lost through a wall depends on its U-value. The wall in the example has a U-value of 1.6 W/m²K: it does not prevent heat loss well, leading to higher heat loss and energy costs (512 watt-hours is poor). |
| **Plenary**  SUGGESTED TIME:  5 minutes  RESOURCES:   * L4 Slide deck – slides 23–27 | * Show students the multiple-choice questions on slides 23–26 and give them one minute to answer each question. * Review the responses provided by students, correcting any misconceptions, and consolidate the learning from the lesson using slide 27. |
| **Follow-up/ consolidation** (to be completed outside of lesson)  SUGGESTED TIME:  30 minutes  RESOURCES:   * L4 Slide deck – slides 28 * L4 Consolidation Worksheet * L4 Consolidation Worksheet answers | * Unless students have already been able to complete their practical report write-up in the lesson, they should complete this as a consolidation task. * As an alternative or additional consolidation task, there is an opportunity for students to practise more calculations involving integration using the Consolidation Worksheet. Fully worked solutions are available so students can mark their own work, or they can be used for peer assessment at the start of the next lesson. |

**Weblinks and resources**

All weblinks and resources have been used in accordance with the owner Terms and Conditions and, where specified, permission has been granted by the owner. Although content has been reviewed, accessibility in externally linked resources cannot be guaranteed.

Inclusion of weblinks or mention of external organisations within the teaching materials does not imply any affiliation with, endorsement by or sponsorship of these external organisations.

|  |  |  |  |
| --- | --- | --- | --- |
| **Location** | **Link**  (with permission if required) | **Owner** | **Date last accessed** |
| TG, page 3 | <https://qualifications.pearson.com/en/qualifications/t-levels/design-surveying-and-planning-for-construction-2025.html> | Pearson\* | September 2025 |
| TG, page 3 | [www.technicaleducationnetworks.org.uk](http://www.technicaleducationnetworks.org.uk) | Technical Education Networks | September 2025 |
| TG, page 6 | [www.bbc.co.uk/bitesize/subjects/z38pycw](http://www.bbc.co.uk/bitesize/subjects/z38pycw) | BBC Bitesize | September 2025 |
| TG, page 6 | [www.nrich.maths.org](http://www.nrich.maths.org) | University of Cambridge | September 2025 |
| TG, page 6 | [www.desmos.com](http://www.desmos.com) | Desmos | September 2025 |
| TG, page 6 | [www.geogebra.org](http://www.geogebra.org) | GeoGebra | September 2025 |
| TG, page 6 | [www.thenational.academy/teachers/key-stages/ks4/subjects/maths/programmes](http://www.thenational.academy/teachers/key-stages/ks4/subjects/maths/programmes) | Oak National Academy | September 2025 |
| TG, page 6 | <https://math.libretexts.org/Courses/Mount_Royal_University/Calculus_for_Scientists_II/2%3A_Techniques_of_Integration/2.5%3A_Numerical_Integration_-_Midpoint%2C_Trapezoid%2C_Simpson's_rule> | LibreTexts | September 2025 |
| TG, page 6 | [www.sketchandcalc.com](http://www.sketchandcalc.com) | SketchAndCalc | September 2025 |
| TG, page 6 | [www.designingbuildings.co.uk/U-values](http://www.designingbuildings.co.uk/U-values) | www.designingbuildings.co.uk | September 2025 |
| TG, page 6 | [www.rockwool.com/uk/resources-and-tools/tools/u-value-calculator/](http://www.rockwool.com/uk/resources-and-tools/tools/u-value-calculator/) | Rockwool | September 2025 |
| TG, page 8 | nucinkis-lab.cc.ic.ac.uk/HELM/helm\_workbooks.html | University of Loughborough | September 2025 |
| TG, page 9 | [www.mathsisfun.com/calculus/](http://www.mathsisfun.com/calculus/) | mathsisfun.com | September 2025 |
| TG, page 9 | [www.khanacademy.org/math/calculus-1](http://www.khanacademy.org/math/calculus-1) | Khan Academy | September 2025 |
| TG, page 9 | [www.mathsisfun.com/calculus/integration-introduction.html](http://www.mathsisfun.com/calculus/integration-introduction.html) | mathsisfun.com | September 2025 |
| TG, page 9 | [www.khanacademy.org/math/integral-calculus/ic-integration](http://www.khanacademy.org/math/integral-calculus/ic-integration) | Khan Academy | September 2025 |
| TG, page 9 | nucinkis-lab.cc.ic.ac.uk/HELM/helm\_workbooks.html | University of Loughborough | September 2025 |
| TG, page 14 | [www.hs2.org.uk/building-hs2/cuttings-and-embankments/](http://www.hs2.org.uk/building-hs2/cuttings-and-embankments/) | HS2 | September 2025 |
| TG, pages 17, 23 | [www.geogebra.org/m/WAKyrVkq](http://www.geogebra.org/m/WAKyrVkq) | GeoGebra | September 2025 |
| TG, page 26 | [www.desmos.com/calculator](http://www.desmos.com/calculator) | Desmos | September 2025 |
| TG, page 26 | [www.geogebra.org/calculator](http://www.geogebra.org/calculator) | GeoGebra | September 2025 |
| TG, page 27 | [www.bwf.org.uk/wp-content/uploads/Future-Home-Standards-U-Value-guidance-document-March-21.pdf](http://www.bwf.org.uk/wp-content/uploads/Future-Home-Standards-U-Value-guidance-document-March-21.pdf) | British Woodworking Federation | September 2025 |
| L3 Consolidation Worksheet 1 | [www.designingbuildings.co.uk/U-values](http://www.designingbuildings.co.uk/U-values) | Designing Buildings | September 2025 |

\*T Level Technical Qualification is a qualification approved by IfATE

**Terms of use and disclaimer of liability**

These resources are made available subject to the Technical Education Networks programme Terms and Conditions. These can be accessed at: [www.technicaleducationnetworks.org.uk](http://www.technicaleducationnetworks.org.uk)

The Terms and Conditions set out the legal terms and associated information relating to the teaching materials and other assets produced as part of the Technical Education Networks programme. The Terms and Conditions may be updated from time to time; please ensure you have read and understood them each time you access the resource.